

Distribution Network Power Loss Minimization by Optimal DG Allocation

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Abstract: This paper presents a methodology based on analytical approach to optimally allocate (site and size) the Distributed Generation (DG) units(s) in radial power distribution networks for minimizing the real power losses. The proposed method requires only the results of base case load flow to determine the optimal size of DG unit(s) required at each bus. For this, suitable analytical expressions have been proposed to determine the optimal size of DG unit(s) to cause total minimum real power loss in a given distribution network with their corresponding optimal location(s). Two cases with two scenarios comprising of DG type and number of DG units (single and multiple), respectively, are considered. The proposed method is applied to an IEEE 33-bus radial distribution test system. Results obtained by this proposed method validate the suitability and importance of appropriate DG allocation and also the number of DG units in power distribution networks.

Keywords: Power Distribution System, Distributed Generation, Sizing, Siting, Real Power loss minimization.

I. Introduction

Electrical power systems are seeing transition from large centralized generation plants connected to the bulk transmission network into the decentralized systems with small generating systems connected directly to distribution networks, near demand centres. The later type of generation system is known as Distributed Generation (DG) [1]. DG can be powered by renewable energy sources (e.g. solar, wind, small hydro, biomass, geothermal, etc.) or non-renewable energy sources (e.g. gas turbine, micro turbine, fuel cell, reciprocating engine, etc.). The potential benefits of DG include: reduction of power losses, improvement in the voltage profile, deferred network expansion cost, network reliability improvement, etc. [2-5].

Appropriate size and location of DG offers technical, economical and environmental benefits to distribution networks. For optimal allocation of DG in distribution networks, different objectives such as power loss minimization [6-8, 12-26], improvement of voltage profile [6, 13, 23, 26], network investment cost minimization [4, 9, 10], reduction of environmental impact [6], etc. were proposed by researchers using single or multi objective problem formulation. Different optimization techniques like probabilistic based mixed integer non-linear programming (MINLP) [14], Analytical approaches [11, 12, 15-18, 23, 24], Genetic algorithm (GA) [10, 19, 24], Artificial bee colony [20], Particle swarm optimization [21], Evolutionary programming [22], GA and Tabu search (GA-TS) [25], Optimal power flow (OPF) [8], heuristic [9] and index based [6, 26] have been applied to solve the DG allocation issues.

Most of the analytical approach based methodologies [12, 15-18, 23, and 24] for optimal allocation of DG in distribution networks considered the DG type which is capable to supply real power only to the network. But, there are other types of DG which can supply real and/or reactive power into the network (rarely formulated) and reduce the power loss considerably and improve the performance to still better extent. Further, majority of the general analytical approaches for DG siting and sizing are based on exact loss formula and require the determination of the bus impedance matrix (Z_{bus}) and its inverse (Y_{bus}^{-1}) or Jacobian matrix, which are computationally demanding. Therefore, due to size, complexity and the specific characteristics of the distribution network, the above methods are not suitable. Hence, the optimal allocation of DG of any type using suitable solution technique needs further attention. Moreover, the proposed method requires only the results of base case load flow to determine the optimal size of DG unit(s) required at each bus.

This paper is organized as follows: Section II discusses the proposed methodology, Section III presents the solution methodology, and Section IV presents the results and discussion of the proposed work. Finally, in Section V, conclusions are summarized.

II. The Methodology Proposed

In this section the mathematical formulation of the proposed analytical approach, to determine the DG size required at various buses of a given distribution network and hence, the appropriate size and location to obtain minimum total real power loss is presented. The following assumptions are made in the analysis of the proposed work:

1. The given network is balanced.
2. The power factor of DG is specified.
3. Voltage variation due to DG placement is negligible.

The objective is to minimize, P_L (1)

$$\text{Subject to, } V_k^{\min} \leq V_k \leq V_k^{\max}$$

The total real power loss in a radial distribution network without DG is given as:

$$P_L = \sum_{i=1}^b I_i^2 R_i = \sum_{i=1}^b (I_{ai}^2 + I_{ri}^2) R_i \quad (2)$$

Where, I_i is the current of the i^{th} branch with I_{ai} and I_{ri} being its real and imaginary components, respectively; and R_i is the resistance of the i^{th} branch. b is the total number of branches in the system. V_k is the voltage magnitude of the DG connected bus, k . V_k^{\min} and V_k^{\max} are the minimum and of voltages at bus k (i.e. 0.95 and 1.05), respectively.

Let a DG be connected at bus k (Fig. 1) injecting current, I_{DG}^k into the network is given as:

$$I_{DG}^k = I_{aDG}^k + jI_{rDG}^k = I_{aDG}^k (1 + j \tan \phi^k) \quad (3)$$

Where, I_{aDG}^k and I_{rDG}^k are the real and reactive components, respectively of I_{DG}^k and ϕ^k is the phase angle of I_{DG}^k

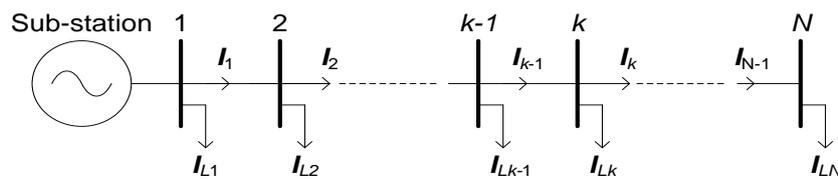


Fig. 1: A typical n-bus radial distribution network

where in fig. 1, n is total number of nodes in the network; I_1 through I_{n-1} are the branch currents before the DG placement; $I_{1DG}^k, \dots, I_{kDG}^k$ through $I_{(n-1)DG}^k$ are the branch currents after DG placement; I_{L1}, \dots, I_{Lk} through I_{Ln} are the load currents at different nodes (1 through k); SS is the substation.

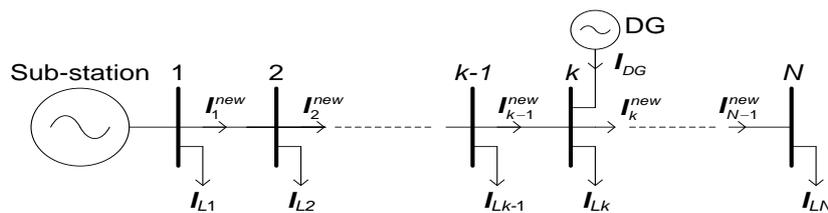


Fig. 2: A typical n-bus radial distribution network

When the DG is connected at bus k in a network (Fig. 2), the real and reactive components of currents in the branches connected between source to DG connected bus only get changed without affecting the currents in the other branches of the network if the change in voltage is assumed to be negligible.

$$\begin{aligned} I_i^{new} &= I_i - D_i \cdot I_{DG}^k \\ &= (I_{ai} - D_i \cdot I_{aDG}^k) + j(I_{ri} - D_i \cdot \tan \phi^k \cdot I_{aDG}^k) \end{aligned} \quad (4)$$

Where, $D_i = \begin{cases} 1, & \text{if branch } i \text{ is between bus 1 (sub - station bus) and bus } k \\ 0, & \text{otherwise.} \end{cases}$

Now, the total real power loss after DG connected at bus k is given by:

$$\begin{aligned}
 P_L^{new} &= \sum_{i=1}^{N-1} (I_i^{new})^2 \cdot R_i \\
 &= \sum_{i=1}^{N-1} \left[(I_{ai} - D_i \cdot I_{aDG}^k)^2 + (I_{ri} - D_i \cdot \tan \phi^k \cdot I_{aDG}^k)^2 \right] \cdot R_i \quad (5)
 \end{aligned}$$

For total real power loss to be minimum, the partial derivative of equation (4) with respect to I_{aDG}^k yields:

$$\frac{\partial P_L^{New}}{\partial I_{aDG}^k} = -2 \sum_{i=1}^b \left[(I_{ai} - D_i I_{aDG}^k) + (I_{ri} - D_i I_{aDG}^k \tan \phi^k) \tan \phi^k \right] D_i R_i \quad (6)$$

The above equation equated to zero and upon simplifying the active part of the DG injected current at bus k is given as:

$$I_{aDG}^k = \sum_{i=1}^b (I_{ai} + I_{ri} \tan \phi^k) D_i R_i / \left(\sum_{i=1}^b (1 + \tan^2 \phi^k) D_i^2 R_i \right) \quad (7)$$

Using (3) the reactive component of DG current can be found and hence, the DG capacity required at bus k is given by:

$$S_{DG}^k = V_k I_{aDG}^k \sqrt{(1 + \tan^2 \phi^k)} \quad (8)$$

From equation (8), it is clear that for the total system real power loss to be a minimum, the optimal size of DG that should be connected at bus k must be S_{DG}^k

The following are the four possible cases of DG types:

Case 1: DG is injecting both real and reactive power (lagging power factor operation), e.g. Synchronous generator

Case 2: DG is injecting real power only (unity power factor operation), e.g. Fuel cell

Case 3: DG is injecting real power and absorbing reactive power, e.g. Induction generator, and

Case 4: when DG injecting reactive power only, e.g. Synchronous condenser.

Only type 1 and 2 DG are considered in this study.

III. The Solution Algorithm

This section presents the solution methodology for the method proposed to minimize the total real power loss by optimally sizing and siting the DG unit in distribution network. Two scenarios are considered as discussed in the next section. The solution algorithm is presented as below.

3.1 Scenario 1: Real power loss minimization by Single DG allocation

The computational steps involved in finding the optimal size and location of DG to minimize the total real power loss in a radial power distribution system are:

1. Run the base case (without DG) load flow using [28] and obtain the branch currents and total real power loss ($Loss_b$) in the given network..
2. Select DG power factor
3. Select a bus (one at a time), except the source bus and find the DG size in terms of real and reactive components of DG injected current using eqns. (7) and (8).
4. Set bus count as $k=2$, place the DG at bus k with the corresponding DG size found in step (2) and calculate the total system real power loss (say, $Loss_k$).
5. Check for the voltage constraint. If voltage constraint is satisfied go to next step, otherwise discard that particular solution
6. Store the values of $Loss_k$.
7. Is this last count? If yes go to step (8), otherwise increase the bus count by 1 and repeat step (3)-(6).
8. Finally, sort the $Loss_k$ values stored and arrange them in the ascending order and select the least one.
9. The DG size and its corresponding location in a given network which results in minimum possible total real power loss gives the optimal size and location, respectively.

The above algorithm provides the optimal DG size and location for a given load level.

3.2 Scenario 2: Real power loss minimization by multiple DG Allocation

In this section, the procedure of single DG allocation extended for allocating multi DG units is presented. In general, the total real power loss in presence of DG units is given by:

$$P_{lossDG} = \sum_{i=1}^b (I_{ai} - \sum_{j=1}^k D_{ij} I_{aDGj})^2 \quad (9)$$

Where k is the number of DG connected buses. I_{DG} , the k -dimensional vector consisting of DG injected currents; α_j the set of branches from the source bus to the j th DG unit bus ($j=1, 2, \dots, k$); the elements of D are

$$D_{ij} = \begin{cases} 1; & \text{if branch } i \in \alpha_j \\ 0; & \text{otherwise.} \end{cases} \quad (10)$$

Let us assume that only three DG units ($k=3$) are to be connected at buses 6, 14 and 29, respectively. For total real power loss to be minimum, differentiating the above equation w. r. to I_{DG1}, I_{DG2} , and I_{DG3} , respectively, and equating to zero we get the value of active component of DG injected current at buses considered, respectively are:

$$I_{aDG1} = \frac{\sum_{i=1}^b (I_{ai} D_{i1} - D_{i1} D_{i2} I_{DG2} - D_{i1} D_{i3} I_{DG3}) R_i}{\sum_{i=1}^b D_{i1}^2 R_i} \quad (11)$$

$$I_{aDG2} = \frac{\sum_{i=1}^b (I_{ai} D_{i2} - D_{i2} D_{i3} I_{DG3} - D_{i2} D_{i1} I_{DG1}) R_i}{\sum_{i=1}^b D_{i2}^2 R_i} \quad (12)$$

$$I_{aDG3} = \frac{\sum_{i=1}^b (I_{ai} D_{i3} - D_{i3} D_{i1} I_{DG1} - D_{i3} D_{i2} I_{DG2}) R_i}{\sum_{i=1}^b D_{i3}^2 R_i} \quad (13)$$

Once the active component of DG injected current are known from equations (11), (12) and (13), the resulting net DG injected current and hence the optimal capacities of each DG units can be obtained by equation (3) and (8), respectively.

Algorithm to allocate multiple DG units: In the presented work, only two DG units has been considered, but, the developed algorithm can be used to allocate any number of DG units as given below:

1. After allocating the first DG unit following the steps given in the section 3.1, update the branch current and bus voltage values and repeat steps (2) - (8) to obtain the size and site for the next DG allocation.
2. Place the DG capacity obtained in step (1) at the corresponding bus and perform the load flow and obtain the real power loss, say, $Loss_{k1}$.
3. Check whether this new power loss ($Loss_{k1}$) is less than that one obtained with single DG scenario (i.e. $Loss_k$) and ensure that voltage constraint is within the statutory limits of 0.95 - 1.05.
4. If yes, consider the second DG allocation, otherwise discard it. Steps (1)-(4), above are repeated for any further DG allocation.

IV. Results and Discussions

The proposed methodology is implemented in Matlab environment, and tested on an IEEE 33-bus test system given in fig. 3. This is a radial distribution system with the total system load demand of 3.72 MW and 2.3 MVAR. The line and load data are taken from [27]-[29]. Before the placement of DG unit (base case) the total real power loss in the given network was 211.2 KW.

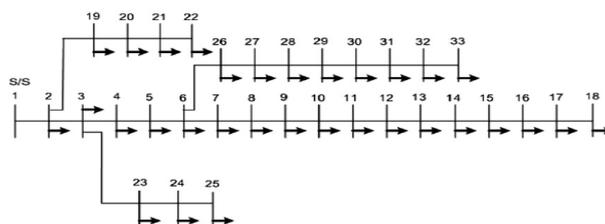


Fig. 3: Single line diagram of an IEEE 33- bus test system

4.1. Scenario 1 : Single DG allocation

The DG power factor is selected equal to the combined system load demand power factor for the possible minimum real power loss and is 0.85 [17]. The optimal size of the DG and the optimal location bus obtained by the proposed method for Case 1 are 2.53 MW and 6, respectively. The total system real power loss with optimal DG at optimal location was found to be 67.97 KW which results in the net percentage real power loss reduction of 67.80. Fig. 4 shows the real power loss and corresponding DG capacity required at different buses of a test network for Case 1, respectively. For the Case 2, the optimal DG size and location are found to be 2.60 MW and 6, respectively. The total system loss with this DG type is 110.4 KW, which results in net percentage real power loss reduction of 47.73. Fig. 4 shows the real power loss and the corresponding DG capacity required at different buses of a test network for Case 2, respectively. The results obtained by the proposed technique are summarized in the table I.

Table I. Comparison of power Loss in different cases

Case	DG Size	Bus	Power Loss (KW)	% Loss reduction
Base case	-----	---	211.20	-----
Case 1	2.53 MW	6	67.97	67.80
Case 2	2.60 MW	6	110.40	47.73

From the table I, it can be observed that more real power loss reduction is possible when DG of optimal size supplying both real and reactive power (i.e. case 1) is allocated at optimal location as compared to the case when DG supplying real power alone (Case 2).

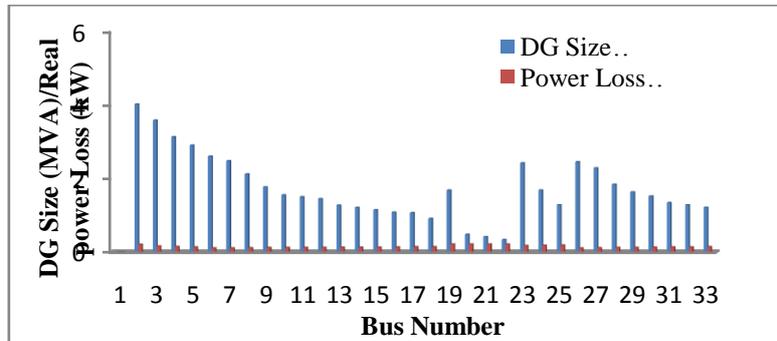


Fig. 4: DG capacity/power losses at different buses for (Case 1)

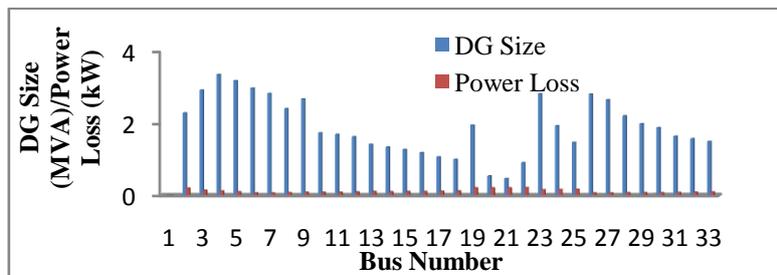


Fig. 5: DG capacity/power losses at different buses for (Case 2)

The other important benefit of proper DG allocation is the improvement in the system voltage profile. The variation of voltage profile at different buses before and after DG placement is shown in fig. 6. Before the DG placement the minimum voltages was at bus 18 and is improved from 0.9065 p.u to 0.9588 p.u, in Case 1 and 0.94134 p.u in Case 2, respectively, when the optimal DG were placed at optimal bus. Further, it can also be observed from the figure that there is significant improvement in the voltage profile at all other buses of the network and the voltages at various buses are within their statutory limits.

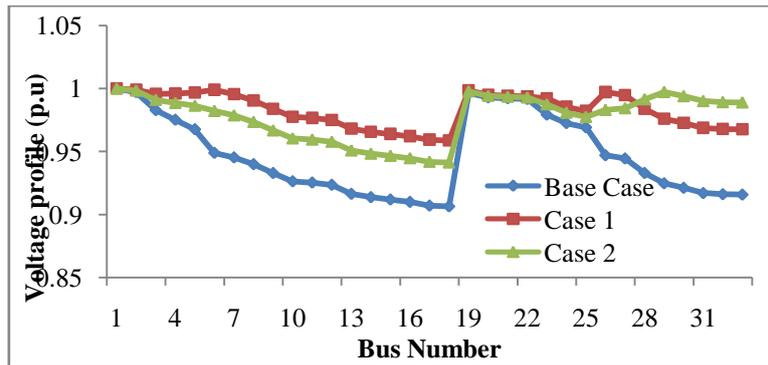


Fig. 6: Voltage profile variation before and after DG placement

To compare the results obtained by the proposed method with the other methods reported in the literature for single DG allocation on a 33-bus test network, the methods proposed in reference [16], [24] and [29] was considered and the results are given in Table II

Table II Comparison of optimal DG unit results

Particulars	[16]	[24]	[20]	Proposed method
Optimal location	6	6	6	6
DG Capacity (MW)	2.49	2.4	2.38	2.53
Percentage Loss reduction	47.33	48.19	44.83	67.80

In the above Table, the results obtained by the proposed method where DG supplies both real and reactive power (i.e. Case 1) were compared with other methods. The DG unit in [16], [24] are limited to supply real power only whereas; the DG unit in [20] is capable to supply both real and reactive power. From the Table, it is clear that more loss reduction is possible by the proposed method compared to the other methods considered.

4.2. Scenario 2 : Multiple DG allocation

Following the algorithm discussed in section 3.1 and 3.2 the results of real power loss reduction and voltage profile improvement for this scenario is presented in this section. Table III compares the results of real power loss reduction obtained by allocating single and multiple (two) DG units in the given test network. Only the DG units with reactive power supplying capability in addition to real power (i.e. Case1) have been considered in the presented analysis.

Table III. Comparison of performance of single/multiple DG scenarios

Particulars	DG Size (MW)	Bus No.	Total real Power Loss (KW)	% Loss reduction
Base case	-----	---	211.20	-----
Single DG	2.53	6	67.97	67.80
Two DGs	1.73	6	50.05	76.30
	0.62	14		

It is seen from table III that when single DG of capacity 2.53 MW (operating at lagging fixed power factor of 0.85) capable to supply real and reactive power is placed at bus 6; the total % real power loss reduction of 67.80 was found. Alternatively, when two small size DG units each of capacity 1.73 and 0.62 MW (both operating at 0.85 pf lag) are placed at bus 6 and 14, respectively, the total real power loss reduction of 76.30 % is obtained. Hence, it can be concluded that use of multi DG units of small optimal size, placed at the optimal location can results in more loss reduction with less total DG capacity compared to single DG scenario.

Comparison of voltage profile between base case (without DG), single and multiple DG scenarios, considering Case 1 only is shown in fig. 7, from which it can be observed that optimal allocation of multiple DG units causes flat voltage profile besides satisfying voltage at various buses are lying within the statutory limits of 0.95 and 1.05 p.u. Before the placement of the DG in the network, the minimum voltage at bus 18 was 0.9065 p.u. After the placement of single and two DG units of optimal size at their respective optimal location bus, this voltage is improved to 0.9588 and 0.9862 p.u, for single and two DG unit scenario, respectively. Therefore, the application of multi DG units of small capacities with reactive power supplying capability can further minimize the total real power loss to 8.5 kW, and improves the voltage profile of the power distribution network significantly compared to single DG scenario.

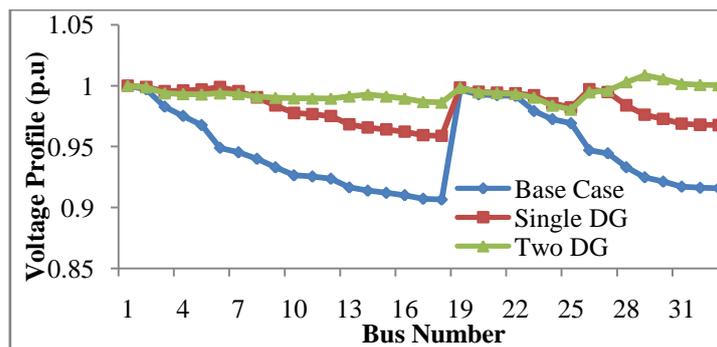


Fig. 7: Comparison of voltage profile of two considered scenarios

V. Conclusions

The integration of DG units into the existing power distribution systems worldwide is increasing and their contribution in the future power system is expected to be even more. A methodology based on the analytical approach for real power loss minimization in power distribution system by optimal sizing and siting of DG unit(s) has been presented, considering DG injecting both real and reactive power with a constant power factor set equal to the combined system load power factor. Results obtained by this proposed method shows better loss reduction, which in turn, results in economic benefit to the utility as well as voltage profile improvement which results in stable operation of the system. In scenario 2, the concept of single DG allocation is extended to multi DG allocation and the obtained results shows significant loss reduction and voltage profile improvement over the single DG scenario.

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